A particle's position in 3D can be written as 3 eqns.

$x(t) = 4t$

$y(t) = 3$

$z(t) = 6t^2 + 2$

Or as one vector equation

$\vec{r}(t) = 4t\,\hat{x} + 3\hat{y} + (t^2 - t - 2)\,\hat{z}$

$(t+1)(t-2)$

Locate the particle at $t = 3$.

Determine its velocity at $t = 1$. Its speed too.

Determine its acceleration.

Find when the particle crosses $z = 0$. $(t = 2)$

Find when the particle changes direction in z. $(t = 1/2)$

Max/min of position = Zero of velocity

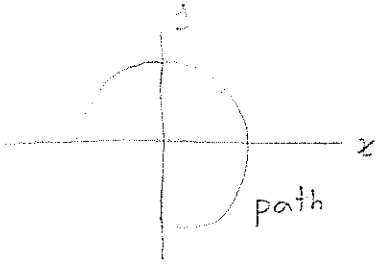
Find when the particle's speed is 10.

$\vec{v}(t) = 4\hat{x} + 0\hat{y} + (2t - 1)\hat{z}$

$|v(t)| = \sqrt{4^2 + (2t-1)^2} = 10$

$16 + (2t-1)^2 = 100$

$t = \frac{1}{2}\left(\sqrt{100-16} + 1\right)$



$$\vec{v} = (5\,m/s)\hat{x} - (5\,m/s)\hat{y}$$

Where is the particle? Two cases: clockwise, counterclockwise.

$\boxed{\text{Projectiles}}$ — horizontal and vertical motion treated separately

Horizontal: $\boxed{x(t) = x(0) + v_x(0)t}$

$v_x(t) = v_x(0)$

$a_x(t) = 0$

from Kinematic Eqns. for constant a.

Vertical: $\boxed{y(t) = y(0) + v_y(0)t - \frac{1}{2}gt^2}$

$v_y(t) = v_y(0) - gt$

$a_y(t) = -g$

$v_y^2(t) = v_y^2(0) - 2g[y(t) - y(0)]$ ← no explicit time

Two ways to write launch velocity:

$v(0) = \sqrt{v_x^2(0) + v_y^2(0)}$

$\theta = \arctan v_y(0)/v_x(0)$

$v_x(0) = v(0)\cos\theta$

$v_y(0) = v(0)\sin\theta$

When launch height = landing height, gunner's eqn.

$y(t) = y(0)$

$y(t) - y(0) = v_y(0)t - \frac{1}{2}gt^2 = 0$

① $v(0)\sin\theta\, t - \frac{1}{2}gt^2 = 0 \Rightarrow t = \dfrac{2v(0)\sin\theta}{g}$

Range $R = x(t) - x(0) = v_x(0)t = v(0)\cos\theta\, t$

$$= \frac{2v^2(0)}{g}\sin\theta\cos\theta = \frac{v^2(0)}{g}\sin 2\theta$$

What angle do a aim at for max range?

My catapult fires at v_0. What angle should I am at to strike the ground R away?

$$R = \frac{v_i^2(0)}{g} \sin 2\theta$$

$$\theta = \frac{1}{2} \arcsin \left(\frac{gR}{v^2(0)} \right)$$

How does this show that there are some ranges my catapult can't reach? (Arg. of arcsin > 1 not allowed.)

Suppose I fire with know initial velocity. $\vec{v} = v_x(0)\hat{x} + v_y(0)\hat{y}$.
How high is the projectile over a spot d away?

Key: Find the time when this occurs.

$$x(t) = x(0) + v_x(0)t$$

$$t' = \frac{x(t) - x(0)}{v_x(0)}$$

$$y(t') = \cancelto{0}{y(0)} + v_y(0)t' - \tfrac{1}{2}gt'^2$$

How fast is it moving as it passes over the spot?

$$v(t') = \sqrt{v_x^2(t') + v_y^2(t')}$$

$$v_y(t') = v_y(0) - gt'$$

$$v_x(t') = v_x(0)$$

Uniform Circular Motion

$a_c = \frac{v^2}{r}$ ← faster circle
← longer rope

I'm spinning a quarter on a coat hanger. The quarter sits about 40 cm from my finger, the center of its loop. How fast do I have to spin to keep the quarter on?

$a_c \geq g$

$\frac{v^2}{r} \geq g$

$v \geq \sqrt{gr} = \sqrt{(10 m/s^2)(0.4 m)} = 2 m/s$

How long does it take to make a complete circle?

$T = \frac{2\pi r}{v}$ ← distance $\approx 1.5 s$

How many cycles per second?

$\frac{1}{T} = \frac{2}{3} Hz$

$\vec{F} = m\vec{a}$

From recording a particle's position over time, infer the force it is moved by. Its mass is also known.

$\vec{r}(t) = t^2\hat{x} + (4t+1)\hat{y} + (8t^3 - 6t)\hat{z}$

$\vec{v}(t) = 2t\hat{x} + 4\hat{y} + (24t^2 - 6)\hat{z}$

$\vec{a}(t) = 2\hat{x} + 48t\hat{z}$

$\vec{F}(t) = m\vec{a}(t) = m(2\hat{x} + 48t\hat{z})$

What force does each rope in a swing exert on a person resting on it.

$v = 0 \Rightarrow a = 0 \Rightarrow F_{net} = 0$

$F_{net} = F_g + F_{ropes} = F_g + 2F_{rope}$

$F_g = ma = -mg$

$F_{rope} = \frac{1}{2}mg$

A climber walks down a wall at a constant speed. What force does the rope exert to enable this safe decent?

$v = \text{constant} \Rightarrow a = 0 \Rightarrow F_{net} = 0$

$F_{net} = F_g + F_{rope}$

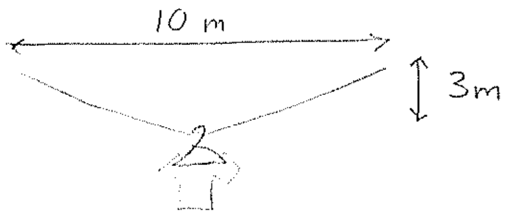
$F_{rope} = mg$

Now the climber is accelerating a little as she goes, at some small a.

$F_{net} = -ma = F_g + F_{rope} = -mg + F$

$F = m(g - a)$ Check $a=0$ limit.

A taut 10 m clothesline sags 3m under the weight of a shirt. What is the tension in the line?

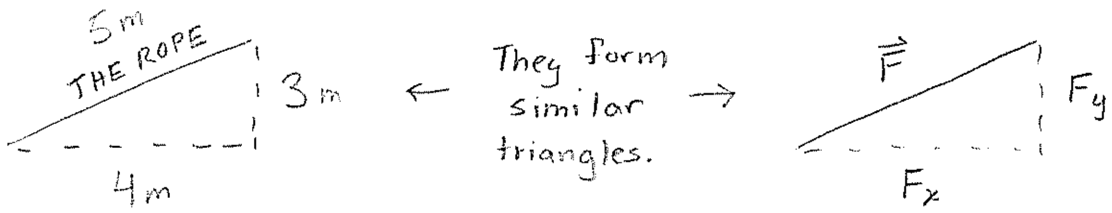


Think about each section of the line alone — say, the right side.

Each side must support $\frac{1}{2}$ of the rope's weight to balance F_g.

$F_y = \frac{1}{2} mg$

* The total force $\vec{F}$ exerted by each section of line must be directed along the line itself. So, although we only really need a force F_y along y, we get a force F_x along x so that the direction of this tension force $\vec{F}$ is pointed along the rope.



$$\frac{F_x}{F_y} = \frac{4}{3}$$

$$F_x = \frac{4}{3} F_y = \frac{4}{3}\left(\frac{1}{2} mg\right)$$

$$F = \sqrt{F_x^2 + F_y^2} = \frac{5}{6} mg$$

On the left section, it's the same story with $F_x \to -F_x$.