Kinematics equations relate

position $x(t)$,

velocity $v(t) = \frac{d}{dt} x(t)$

over time t

when acceleration $a(t) = \frac{d^2x}{dt^2}$ is constant: $a(t) = a$.

All kinematics problems can be solved by

1. Identifying which variables are known,
2. Choosing the equation (or combination of equations) that relate them, and
3. Solving algebraically.

Virtually always, we have information about at least one important moment. We label time $t = 0$ at the moment. Earlier events occur at some $t < 0$; Following events occur at some $t > 0$.

Examples:

A train passes a signal traveling 10 m/s.

Call the moment it passes $t = 0$.
$v(0) = 10$ m/s

A train starts out from a station and arrives at a station 8 Km away 15 minutes later.

$x(0) = 0$
$x(15) = 8$ Km
$v(0) = 0$

A ball is launched from the ground straight upward at 100 Km/hr

$x(0) = 0$
$v(0) = 100$ Km/hr

$$v(t) = v(0) + at$$

→ no x
a through this time
v at two moments
t between those moments

A ball is lauched upward at v_0. How long until it hits the ground?

$v(0) = v_0$

If it hits the ground at time t, $v(t) = -v_0$.

$-v_0 = v_0 - gt$

Suppose that a cyclist whose bike slows more suddenly than $5\ m/s^2$ is thrown over the handlebars.

A cyclist moving 12 m/s comes to a stop 2 seconds later. Does she stay on the bike?

$v(0) = 12\ m/s$
$v(2) = 0$
$v(2) = v(0) + 2a$
$a > 5\ m/s^2$

Water flowing in an open pipe (think of a waterslide) gains speed at $1\ m/s^2$. If a sample of water is moving at 6 m/s at 12:00, what is its speed at 12:01?

$v(60) = v(0) + 60a$

If a train going 120 Km/hr has been accelerating at $40\ Km/hr^2$ since it began its trip, how long ago did it start?

$120\ Km/hr = 0 + (40 Km/hr^2)t$

$$x(t) - x(0) = \frac{1}{2}\left[v(t) + v(0) \right] t$$

→ no a
x at two moments
v at two moments
t between these moments

A train has been accelerating constantly for a half hour. It is now traveling 135 Km/hr. How far has it come?

A ball tossed up on a strange planet (acceleration of gravity unknown) with velocity v_0 take time t to fall back to the tosser. How high did it go?

$$x(t) - 0 = \frac{1}{2}\left[0 + v_0 \right] t$$

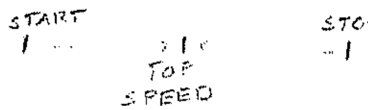
A ball rolls away, gradually coming to a stop after 30 seconds. It rolled 100 meters. When it started rolling away, what was its speed?

$$x(30) - x(0) = 100 \text{ m}$$

$$v(30) = 0$$

$$100 \text{ m} = \frac{1}{2}\left[0 + \underline{v(0)} \right](30 \text{ s})$$

I am driving block by block, stopping at stop signs 100 m apart. I'm in a hurry, but cheese on the hot pizzas in my trunk will slide off if I accelerate more than $\pm 2 \text{ m/s}^2$. How fast can I cover one block?



$$v(t) = (2 \text{ m/s}^2)t$$

$$50 \text{ m} = \frac{1}{2}\left[(2 \text{ m/s}^2)t' + 0 \right] t' \Rightarrow t' = \sqrt{50} \text{ s}$$

$$t = 2\sqrt{50} \text{ s}$$

A firework climbs to 500 m building up to the speed 100 m/s right before it explodes. How long did that take?

$$[v(t)]^2 = [v(0)]^2 + 2a[x(t) - x(0)]$$

a through this time
x at two moments
v at two moments
→ no t

A firework climbs to 500 m, accelerating at 10 m/s² all the way up. What is its velocity at this time?

A ball is launched upward at 100 m/s. Estimating Earth gravity as 10 m/s², how high does it go?

A paratrooper in the 101st Airborne is plummeting at 20 m/s. He pulls his ripcord, and his parachute begins to slow his fall. He has fallen another 100 m by the time his chute slows him to 5 m/s. What was the acceleration enacted by the chute?

$$(5 \text{ m/s})^2 = (20 \text{ m/s})^2 + 2a\,(100 \text{ m})$$

$$x(t) - x(0) = v(0)t + \frac{1}{2}at^2$$
$$x(t) - x(0) = v(t)t - \frac{1}{2}at^2$$

a through this time
x at two moments
→ v at only one moment
t between these moments

A skydiver jumps from a horizontally flying plane. How far has she fallen after a time t of free-fall?

She jump again from a plane flying upword at 50 m/s. How far has she fallen after a time t of free-fall?

If she is 100 m down from her jumping point, how long has she been falling?

Now suppose the plane's velocity is unknown, but she finds herself 100 m down in 10 seconds. What is her velocity now? What was her velocity right before the jump (this is also the plane's velocity)?